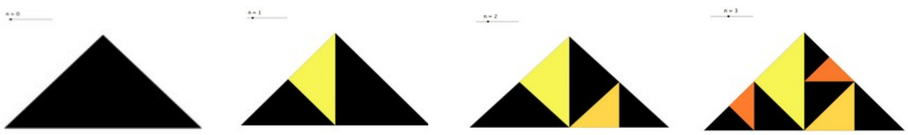
**Free Think Friday: Fibonacci Fun** Bethany Davis

**Problem Statement**

The problem that I chose to work on, along with fellow students, was titled “Fibonacci Fun”. The premise of the problem was based around a continually dividing sequence that was occurring within a triangle. For the problem we were given the first four stages of this sequence as shown below.



n=0 n=1 n=2 n=3

For someone who is unaware of what the Fibonacci sequence is, it is a series of numbers where the next number in the series is found by adding the two numbers that are before it. Before this problem, I can say I had no idea as to what the alliteration at the top of the page meant. Below the diagrams was a list of two tasks in which we were asked to work through. The first task was dealing with the black region. It asked how the area of the black region will change as the pattern progresses if the legs (or the two sides that are not the hypotenuse) of the first triangle are a value of one. The second task asked what the area was of the triangles that are being added as the pattern continues. Before really looking at these tasks, we first began to discuss what the Fibonacci sequence was and how it spirals if graphed. We attempted to visualize this to see if it would give some insight as to what the pattern was actually doing before starting on the problems at hand.

**Process Description**

Going back to the first task, my group's initial reaction was to calculate how much space each new triangle took up in the existing black region. For the first iteration of the pattern, we saw that the triangle took up one quarter of the initial shape. Therefore, the black region decreased by ¼. We continued to find more evidence by looking at the next sequence, and it seemed to be a reasonable conclusion. A week went by before we returned to the problem, and we realized that our first response was incorrect. It did not hold up through the entirety of the problem. We decided to break it down more and calculate how it changes using a denominator that would apply to all three iterations. That of sixteen because as we see in the third iteration, the smallest triangle can fit sixteen of itself into the original triangle. Therefore when n=0, the black region is equal to 16/16, or one. When n=1, the black region is 12/16. The second iteration has a black region area of 10/16, and the third has a black area of 8/16. When we subtract each of these we get a total of 1/8 of the area being removed.

For the second problem, we ended the first week with ideas as to the answer, but we had great difficulty in articulating it in a non ambiguous way. We did not really begin to answer that question fully and only spoke briefly about it for a minute or so before the question was put to rest till next week. Coming into class that next friday, we all saw the error we made in the first one and worked towards finding a more reasonable conclusion. Then we went back to the second task that we had difficulty with the last time. Similar to how we worked on the first one, we wrote down what we knew of the triangles from each sequence and tried to find a concrete pattern that we could describe between them. I do not think we have gotten close enough to be able to state the pattern in a sentence, but we have formed some sort of definitive answer. The areas of the new triangles are one over the number of pieces that makes up the triangle. The denominator is being doubled for each new sequence in the pattern. Such as the first triangle that is ¼ of the total area, and the next, smaller triangle is equal to 1/8. We can see that the first triangle is being multiplied by ½ to get to the next. This continues for third iteration as well.

**Solution**

The solution that our table found for the first task was that the black region is decreasing by 1/8 for every new sequence in the pattern. The reason we know this to at least be partially accurate is because we took into account each iteration given in the original description of the problem. The solution that was found for the second task was that the areas of the new triangles are half the area of the previous triangle. The first triangle is ¼ the area while the next sequence shows a triangle being equal to 1/8 the area of the original triangle. Therefore the area of the triangles are being halved.

**Self-Assessment and Reflection**

First off, I would like to say that I truly do enjoy open-ended, group problems because they are very engaging and can be very informative. Some times much more than standard classwork because they express ideas that are more easily relatable to other math topics or real world scenarios. From this specific problem, I learned more about sequences and searching for them within a problem. I find that being able to think mathematically has helped me just as much as understanding particular concepts within the subject. Therefore, being able to discover patterns and use them to solve problems can be very important. Especially in a problem that you do not know how to start. I personally have found this useful because I have struggled in the past with applying knowledge of a problem to open-ended problems such as these. During this problem I have also seen how others approach problems like this and how the way they view them help us get to the same conclusion. For example, at the beginning a member started to describe the whole picture and what could be happening versus just looking at what was in front of us. In a continuous pattern such as this, it can be helpful to view it as such and give perspective.

I think that I deserver a ten out of ten because of my consistent determination to complete the problem and attempt to find a solution. Although I am not sure of the definitiveness of our conclusion, I do believe that we were on the right track. A mathematical practice and expectation that I believe has greatly shown in our work through the problem is to make sense of a problem and persevere in solving it. It was an interesting problem to figure out, and I often tried to question what we were doing and how we were doing it so that we may be certain that we were going about solving it the right way. Or rather a way that would accumulate the most evidence and make the most sense to an outside party.